

[Weekly Test - 3]

Set A

Q. Trapezoidal Rule $\int_0^1 \frac{x^2}{1+x^3} dx$

Sol. -

Here - $a = 0$, $b = 1$, $n = 10$

$$h = \frac{b-a}{n} = \frac{1-0}{10} = 0.1$$

$$f(x) = \frac{x^2}{1+x^3}$$

x	$f(x) = \frac{x^2}{1+x^3}$	y
0	$\frac{0^2}{1+0^3} = 0$	y_0
0.1	$\frac{0.1^2}{1+0.1^3} = 0.9990$	y_1
0.2	$\frac{0.2^2}{1+0.2^3} = 0.0396$	y_2
0.3	$\frac{0.3^2}{1+0.3^3} = 0.0876$	y_3
0.4	$\frac{0.4^2}{1+0.4^3} = 0.1503$	y_4
0.5	$\frac{0.5^2}{1+0.5^3} = 0.2222$	y_5
0.6	$\frac{0.6^2}{1+0.6^3} = 0.2960$	y_6
0.7	$\frac{0.7^2}{1+0.7^3} = 0.3648$	y_7
0.8	$\frac{0.8^2}{1+0.8^3} = 0.4232$	y_8
0.9	$\frac{0.9^2}{1+0.9^3} = 0.4684$	y_9
1	$\frac{1^2}{1+1^3} = 0.5$	y_{10}

By Trapezoidal Rule

$$f(x) = \frac{x^2}{1+x^3} = \frac{h}{2} \left[(y_0 + y_{10}) + 2(y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9) \right]$$

$$= \frac{0.1}{2} \left[(0+0.5) + 2(0.9990 + 0.0396 + 0.0876 + 0.1503 + 0.2222 + 0.2960 + 0.3648 + 0.4232 + 0.4684) \right]$$

$$= \frac{0.1}{2} (0.5 + 6.1022)$$

$$\frac{0.1}{2} (6.6022) = 0.33011 \text{ Ans.}$$

2

Ans - 3

$$\int_1^5 \frac{dx}{x}$$

Sol. Here

$$a = 1, b = 5, n = 8$$

$$h = \frac{b-a}{n} = \frac{5-1}{8} = 0.5$$

x	$f(x) = \frac{1}{x}$	
1	$\frac{1}{1} = 1$	40
1.5	$\frac{1}{1.5} = 0.6666$	41
2	$\frac{1}{2} = 0.5$	42
2.5	$\frac{1}{2.5} = 0.4$	43
3	$\frac{1}{3} = 0.3333$	44
3.5	$\frac{1}{3.5} = 0.2857$	45
4	$\frac{1}{4} = 0.25$	46
4.5	$\frac{1}{4.5} = 0.2222$	47
5	$\frac{1}{5} = 0.2$	48

1

By Simpson's 3/8 Rule

$$f(x) \frac{dx}{x} = \frac{3h}{8} [(y_0 + y_8) + 2(y_3 + y_6) + 3(y_2 + y_5 + y_4 + y_7)]$$

$$= \frac{3 \times 0.5}{8} [(1 + 0.2) + 2(0.4 + 0.25) + 3(0.6666 + 0.5 + 0.3333 + 0.2857 + 0.2222)]$$

$$\frac{3 \times 0.5}{8} (1.2 + 1.3 + 6.0234)$$

$$\frac{3 \times 0.5}{8} (8.5234) = 1.5981 \text{ Ans.}$$

Ans = 4 $\int_0^\pi \sin x \, dx$

Sol Here $a = 0$, $b = \pi$, $n = 10$

$$h = \frac{b-a}{n} = \frac{\pi - 0}{10} = \frac{\pi}{10}$$

$$f(x) = \sin x$$

x	$f(x) = \sin x$	
$\pi/10 * 0 = 0$	$\sin x = 0$	40
$\pi/10 * 1 = \pi/10$	$\sin x = 0.3090$	42
$\pi/10 * 2 = \pi/5$	$\sin x = 0.5877$	42
$\pi/10 * 3 = 3\pi/10$	$\sin x = 0.8090$	43
$\pi/10 * 4 = 4\pi/10$	$\sin x = 0.9510$	44
$\pi/10 * 5 = \pi/2$	$\sin x = 1$	45
$\pi/10 * 6 = 6\pi/10$	$\sin x = 0.9510$	46
$\pi/10 * 7 = 7\pi/10$	$\sin x = 0.8090$	47
$\pi/10 * 8 = 8\pi/10$	$\sin x = 0.5877$	48
$\pi/10 * 9 = 9\pi/10$	$\sin x = 0.3090$	49
$\pi/10 * 10 = \pi$	$\sin x = 0$	410

By Trapezoidal Rule

$$f(x) \sin x = \frac{h}{2} \left[(y_0 + y_{10}) + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9) \right]$$

$$\frac{\pi/10}{2} \left[(0+0) + 2(0.3090 + 0.5877 + 0.8090 + 0.9510 + 1 + 0.9510 + 0.8090 + 0.5877 + 0.3090) \right]$$

$$\frac{\pi/10}{2} (12.6268) = 1.9834 \text{ Ans.}$$

By Simpson's $\frac{2}{3}$

$$f(x) \sin x = \frac{h}{3} \left[(y_0 + y_{10}) + 2(y_2 + y_4 + y_6 + y_8) + 4(y_1 + y_3 + y_5 + y_7 + y_9) \right]$$

$$\frac{\pi/10}{3} = (6.1548 + 12.944)$$

$$\frac{\pi/10}{3} \approx (19.0988) = 2.0000 \text{ Ans.}$$

12

$\frac{\pi}{10}$ Ans